Optical Waveguide Theory (H)

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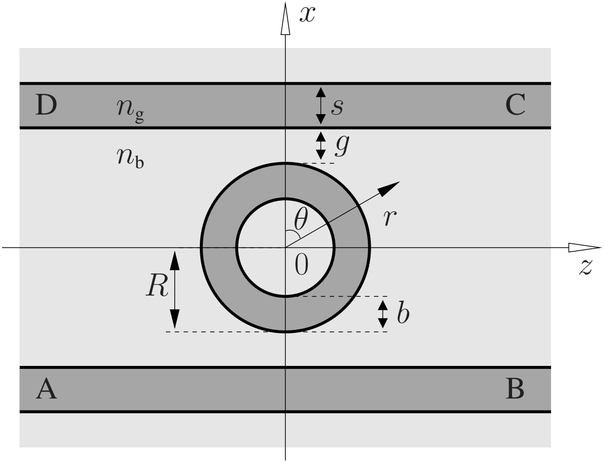
Optical waveguide theory

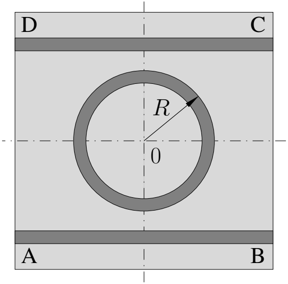
- A Photonics / integrated optics; theory, motto; phenomena, introductory examples.
- B Brush up on mathematical tools.
- C Maxwell equations, different formulations, interfaces, energy and power flow.
- D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.
- E Normal modes of dielectric optical waveguides, mode interference.
- F Examples for dielectric optical waveguides.
- G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
- H Bent optical waveguides; whispering gallery resonances; circular microresonators.
- I Coupled mode theory, perturbation theory.
- • Hybrid analytical / numerical coupled mode theory.
- J A touch of photonic crystals; a touch of plasmonics.
- • Oblique semi-guided waves: 2-D integrated optics.
- • Summary, concluding remarks.

Circular traveling wave resonators



Integrated optical micro-ring or micro-disk resonators.

Ringresonator: Abstract model



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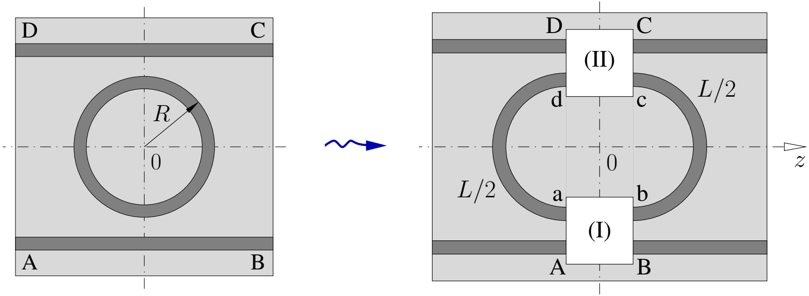

- Ringresonator $\approx$ 2 couplers + 2 cavity segments

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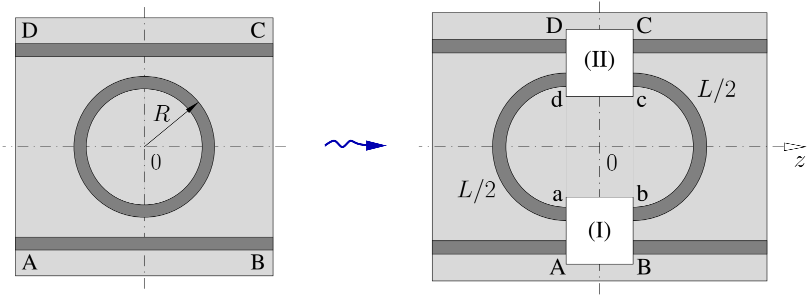
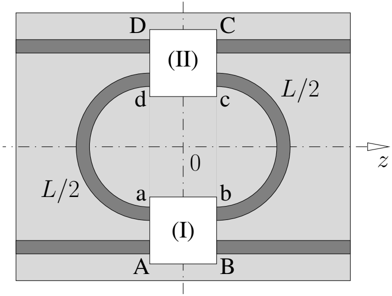


- Ringresonator $\approx$ 2 couplers + 2 cavity segments
- CW description : $\boldsymbol{E}, \boldsymbol{H} \sim \mathrm{e}^{\mathrm{i}\omega t}$, $\omega = k\,\mathrm{c}$, $k = 2\pi/\lambda$.

Couplers: Scattering matrices



- Uniform polarization, single mode waveguides.
- Linear, nonmagnetic (attenuating) elements.
- Backreflections are negligible.
- Interaction restricted to the couplers $\leftrightarrow$ “port” definition.

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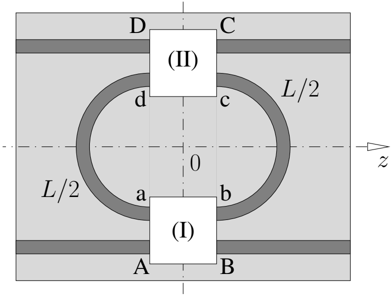

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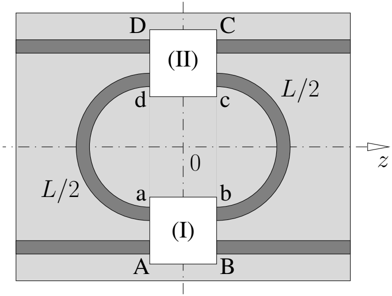
↪ *Symmetric* coupler scattering matrices :

$$\begin{pmatrix} A_- \\ a_- \\ B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & \rho & \kappa \\ 0 & 0 & \chi & \tau \\ \rho & \chi & 0 & 0 \\ \kappa & \tau & 0 & 0 \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \\ B_- \\ b_- \end{pmatrix}$$

$A_\pm,\ B_\pm,\ a_\pm,\ b_\pm$: Amplitudes of waves traveling in $\pm z$-direction.

Coupler symmetries



Symmetry $z \to -z$:

$$A_+ \to b_+ \overset{!}{=} B_- \to a_-$$

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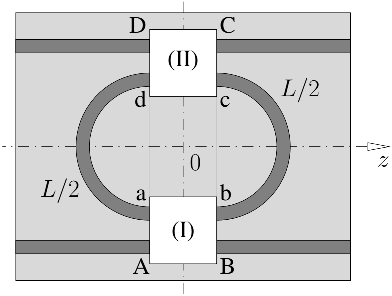

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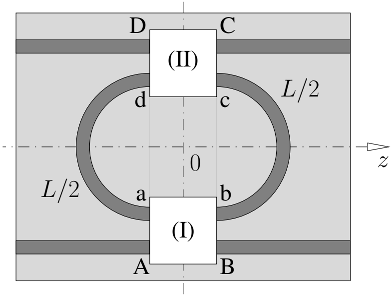

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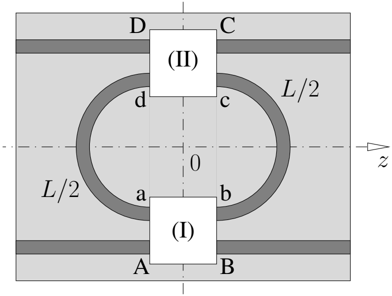

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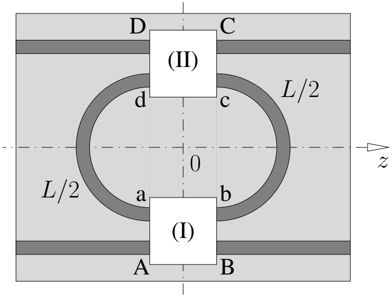
Symmetry $x \to -x$, (I) = (II):

$$\hookrightarrow \begin{pmatrix} D_- \\ d_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} C_- \\ c_- \end{pmatrix}, \qquad \begin{pmatrix} C_+ \\ c_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} D_+ \\ d_+ \end{pmatrix}.$$

Cavity segments

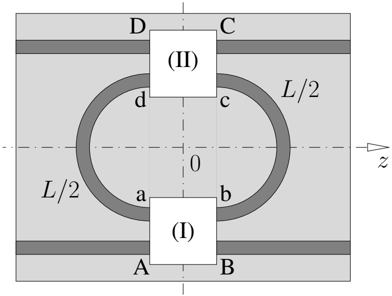


Field evolution $\sim \mathrm{e}^{-\mathrm{i}\gamma s}$
along the cavity core,
propagation distance s.

$\gamma = \beta - \mathrm{i}\alpha$,

β : phase propagation constant,
α : attenuation constant.

(↔ bend modes, to come.)



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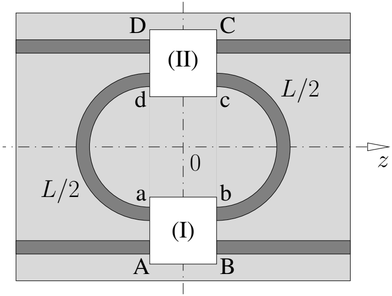
Relations of amplitudes at the ends of the cavity segments :

$$c_- = b_+ \, \mathrm{e}^{-\mathrm{i}\beta L/2} \; \mathrm{e}^{-\alpha L/2}, \qquad a_+ = d_- \, \mathrm{e}^{-\mathrm{i}\beta L/2} \; \mathrm{e}^{-\alpha L/2},$$
$$b_- = c_+ \, \mathrm{e}^{-\mathrm{i}\beta L/2} \; \mathrm{e}^{-\alpha L/2}, \qquad d_+ = a_- \, \mathrm{e}^{-\mathrm{i}\beta L/2} \; \mathrm{e}^{-\alpha L/2}.$$

Output amplitudes



Coupler scattering matrices

+ Cavity field evolution

+ External input amplitudes

$$A_+ = \sqrt{P_{\text{in}}}\,,$$

$$B_- = C_- = D_+ = 0$$

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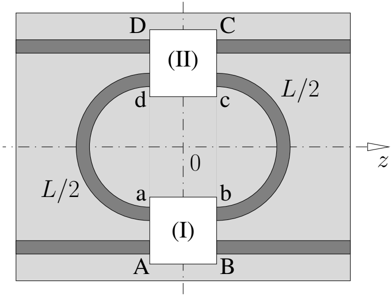

Coupler scattering matrices

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$A_+ = \sqrt{P_{\text{in}}}\,,$

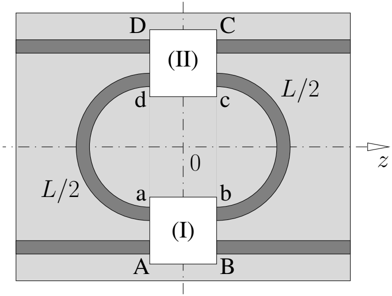
$B_- = C_- = D_+ = 0$

External output amplitudes :

$$A_- = 0, \quad C_+ = 0, \quad D_- = \frac{\kappa^2 p}{1-\tau^2 p^2} A_+\,, \quad B_+ = \left(\rho + \frac{\kappa^2 \tau p^2}{1-\tau^2 p^2}\right) A_+\,,$$

$$p = \mathrm{e}^{-\mathrm{i}\beta L/2}\ \mathrm{e}^{-\alpha L/2}.$$

Power transfer



Power drop : $P_{\mathrm{D}} = |D_-|^2$,

Transmission : $P_{\mathrm{T}} = |B_+|^2$.

$$P_{\mathrm{D}} = P_{\mathrm{in}} \frac{|\kappa|^4\, \mathrm{e}^{-\alpha L}}{1 + |\tau|^4\, \mathrm{e}^{-2\alpha L} - 2|\tau|^2\, \mathrm{e}^{-\alpha L} \cos(\beta L - 2\varphi)}$$

$$P_{\mathrm{T}} = P_{\mathrm{in}} \frac{|\rho|^2 (1 + |\tau|^2 d^2\, \mathrm{e}^{-2\alpha L} - 2|\tau| d\, \mathrm{e}^{-\alpha L} \cos(\beta L - \varphi - \psi))}{1 + |\tau|^4\, \mathrm{e}^{-2\alpha L} - 2|\tau|^2\, \mathrm{e}^{-\alpha L} \cos(\beta L - 2\varphi)}$$

$\tau =: |\tau|\, \mathrm{e}^{\mathrm{i}\varphi}, \;\; d\, \mathrm{e}^{\mathrm{i}\psi} := \tau - \kappa^2/\rho,$

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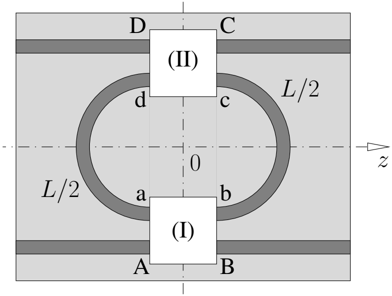

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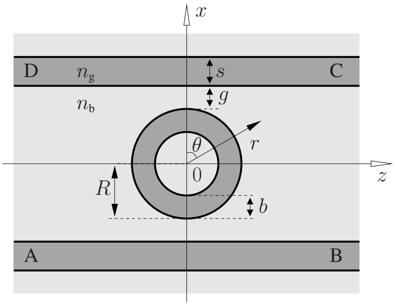
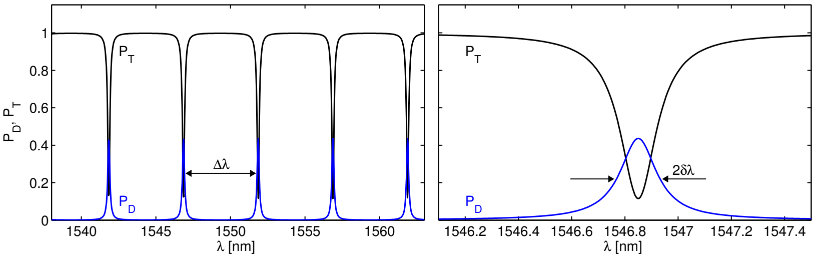
Spectral response



$R = 50\,\mu\text{m},\ b = s = 1.0\,\mu\text{m},\ g = 0.9\,\mu\text{m},$
$n_b = 1.45,\ n_g = 1.60;\ \text{2-D},\ \text{TE}.$

$\Delta\lambda = 5.0\,\text{nm},\ 2\delta\lambda = 0.17\,\text{nm},$
$F = 30,\ Q = 9400,\ P_{D,res} = 0.44.$



Resonances

$$P_{\mathrm{D}} = P_{\mathrm{in}} \frac{|\kappa|^4 \, \mathrm{e}^{-\alpha L}}{1 + |\tau|^4 \, \mathrm{e}^{-2\alpha L} - 2|\tau|^2 \, \mathrm{e}^{-\alpha L} \cos(\beta L - 2\varphi)}$$

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$$\beta = \frac{2m\pi + \phi}{L_{\mathrm{cav}}} =: \beta_m \quad \text{integer } m; \qquad P_{\mathrm{D}}\big|_{\beta=\beta_m} = P_{\mathrm{in}} \frac{|\kappa|^4 \, \mathrm{e}^{-\alpha L}}{(1 - |\tau|^2 \, \mathrm{e}^{-\alpha L})^2} \, .$$

Free spectral range

- Resonance next to β_m :

$$\beta_{m-1} = \frac{2(m-1)\pi + \phi}{L_{\text{cav}}} = \beta_m - \frac{2\pi}{L_{\text{cav}}}$$

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- $\partial_\lambda \beta = ?$
  q_j : waveguide parameters with dimension length,
  $\beta(a\lambda, aq_j) = \beta(\lambda, q_j)/a, \qquad \partial_a \,|_{a=1}$

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FSR : $$\Delta\lambda = -\frac{2\pi}{L_{\mathrm{cav}}}\left(\left.\frac{\partial\beta}{\partial\lambda}\right|_m\right)^{-1} \approx \left.\frac{\lambda^2}{n_{\mathrm{eff}}\,L_{\mathrm{cav}}}\right|_m, \qquad n_{\mathrm{eff}} = \beta/k.$$

(Free spectral range, the spectral distance (here: wavelength) between the drop peaks / the transmission dips).)

Spectral width of the resonances

- $P_{\mathrm{D}} = P_{\mathrm{in}} \dfrac{|\kappa|^4 \, \mathrm{e}^{-\alpha L}}{1 + |\tau|^4 \, \mathrm{e}^{-2\alpha L} - 2|\tau|^2 \, \mathrm{e}^{-\alpha L} \cos(\beta L_{\mathrm{cav}} - \phi)}$,

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- Expansion of cos-terms

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  $$\hookrightarrow \quad \delta\beta = \pm \frac{1}{L_{\mathrm{cav}}} \left(\frac{1}{|\tau|} \, \mathrm{e}^{\alpha L/2} - |\tau| \, \mathrm{e}^{-\alpha L/2} \right) \approx -\frac{\beta_m}{\lambda} \, \delta\lambda$$

FWHM:
$$2\delta\lambda = \frac{\lambda^2}{\pi L_{\mathrm{cav}} n_{\mathrm{eff}}}\bigg|_m \left(\frac{1}{|\tau|} \, \mathrm{e}^{\alpha L/2} - |\tau| \, \mathrm{e}^{-\alpha L/2} \right) .$$

(Full width at half maximum of the spectral drop peaks / the transmission dips (wavelength).)

Finesse & Q-factor

Finesse :
$$F = \frac{\Delta\lambda}{2\delta\lambda} = \pi \frac{|\tau|\,\mathrm{e}^{-\alpha L/2}}{1 - |\tau|^2\,\mathrm{e}^{-\alpha L}}\,.$$

Finesse & Q-factor

Finesse :
$$F = \frac{\Delta\lambda}{2\delta\lambda} = \pi \frac{|\tau|\, e^{-\alpha L/2}}{1 - |\tau|^2\, e^{-\alpha L}} \,.$$

Q-factor :
$$Q = \frac{\lambda}{2\delta\lambda} = \pi \frac{n_{\mathrm{eff}} L_{\mathrm{cav}}}{\lambda} \frac{|\tau|\, e^{-\alpha L/2}}{1 - |\tau|^2\, e^{-\alpha L}} = \frac{n_{\mathrm{eff}} L_{\mathrm{cav}}}{\lambda} F \,.$$

Finesse & Q-factor

Finesse:
$$F = \frac{\Delta\lambda}{2\delta\lambda} = \pi \frac{|\tau|\,\mathrm{e}^{-\alpha L/2}}{1 - |\tau|^2\,\mathrm{e}^{-\alpha L}}\,.$$

Q-factor:
$$Q = \frac{\lambda}{2\delta\lambda} = \pi \frac{n_{\mathrm{eff}} L_{\mathrm{cav}}}{\lambda}\,\frac{|\tau|\,\mathrm{e}^{-\alpha L/2}}{1 - |\tau|^2\,\mathrm{e}^{-\alpha L}} = \frac{n_{\mathrm{eff}} L_{\mathrm{cav}}}{\lambda} F\,.$$

or
$$Q = kRn_{\mathrm{eff}}F \qquad \text{for} \qquad L_{\mathrm{cav}} = 2\pi R\,.$$

Performance versus coupling strength & losses

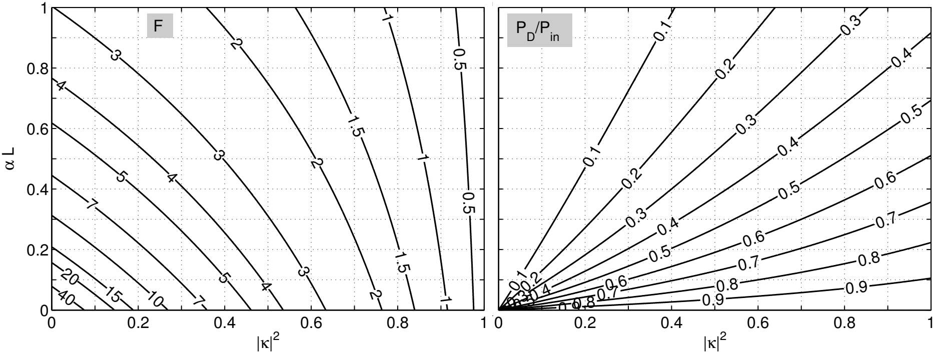
$$\text{Assumption: Lossless coupler elements, } |\rho|^2 = |\tau|^2 = 1 - |\kappa|^2.$$

$$F = \pi \frac{(\sqrt{1-|\kappa|^2})\, \mathrm{e}^{-\alpha L/2}}{1-(1-|\kappa|^2)\, \mathrm{e}^{-\alpha L}}\,, \qquad P_{\mathrm{D}}\big|_{\mathrm{res}} = P_{\mathrm{in}} \frac{|\kappa|^4\, \mathrm{e}^{-\alpha L}}{(1-(1-|\kappa|^2)\, \mathrm{e}^{-\alpha L})^2}\,.$$

Performance versus coupling strength & losses

Assumption : Lossless coupler elements, $|\rho|^2 = |\tau|^2 = 1 - |\kappa|^2$.

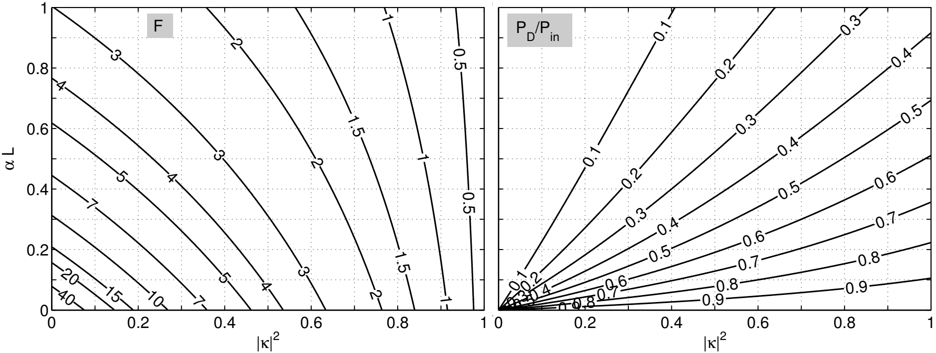
$$F = \pi \frac{(\sqrt{1 - |\kappa|^2})\, \mathrm{e}^{-\alpha L/2}}{1 - (1 - |\kappa|^2)\, \mathrm{e}^{-\alpha L}}\,, \qquad P_{\mathrm{D}}|_{\mathrm{res}} = P_{\mathrm{in}} \frac{|\kappa|^4\, \mathrm{e}^{-\alpha L}}{(1 - (1 - |\kappa|^2)\, \mathrm{e}^{-\alpha L})^2}\,.$$



Performance versus coupling strength & losses

Assumption : Lossless coupler elements, $|\rho|^2 = |\tau|^2 = 1 - |\kappa|^2$.

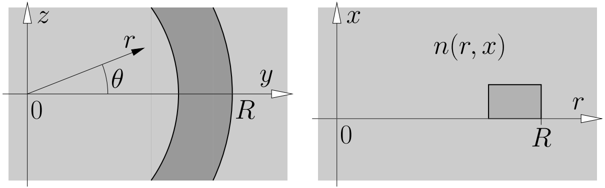
$$F = \pi \frac{(\sqrt{1-|\kappa|^2})\, e^{-\alpha L/2}}{1-(1-|\kappa|^2)\, e^{-\alpha L}}\,, \qquad P_D|_{res} = P_{in} \frac{|\kappa|^4\, e^{-\alpha L}}{(1-(1-|\kappa|^2)\, e^{-\alpha L})^2}\,.$$



$\alpha, \kappa = ?$

Modes of bent waveguides

$\sim \exp(\mathrm{i}\omega t)$ (FD)



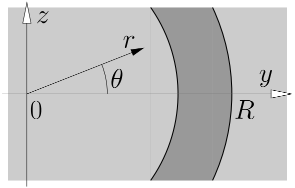
- Constant curvature $\leftrightsquigarrow$ cylindrical coordinates r, θ, x.
- Bend radius R, $\partial_\theta \epsilon = 0$, $\partial_\theta n = 0$

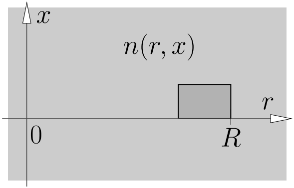
$$\hookrightarrow \quad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r, \theta, x) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(r, x)\, \mathrm{e}^{-\mathrm{i}\gamma R\theta}, \quad \text{bend modes,}$$

$\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}$: bend mode profile, components $\bar{E}_r, \bar{E}_\theta, \bar{E}_x, \bar{H}_r, \bar{H}_\theta, \bar{H}_x$,
$\gamma = \beta - \mathrm{i}\alpha \in \mathbb{C}$: propagation constant,
$\beta \in \mathbb{R}$: phase constant,
$\alpha \in \mathbb{R}$: attenuation constant.

(Exponent $\mathrm{i}\,\gamma R\theta$: a convention, “propagation distance” $R\theta$.)

Modes of bent waveguides





$\sim \exp(\mathrm{i}\omega t)$ (FD)

- Piecewise constant $n(r,x)$, $\psi \in \{\bar{E}_r, \bar{E}_\theta, \bar{E}_x, \bar{H}_r, \bar{H}_\theta, \bar{H}_x\}$,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \left(k^2n^2 - \frac{\gamma^2R^2}{r^2}\right)\psi = 0, \qquad \text{where } \partial n = 0,$$

& continuity conditions at interfaces (cylindrical coordinates),

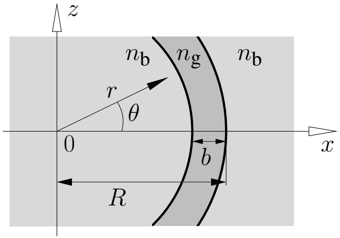
& boundary conditions:
regularity at $r = 0$, outgoing waves at $x = \pm\infty$, $r = \infty$.

(or: normalizability versus x.)

Vectorial 3-D bend mode eigenvalue problem.

(Practical setting: computational domain $r_\mathrm{i} < r < r_\mathrm{o}$, $x_\mathrm{b} < x < x_\mathrm{t}$, PML boundary conditions / $\psi = 0$ at $r = r_\mathrm{i}$.)

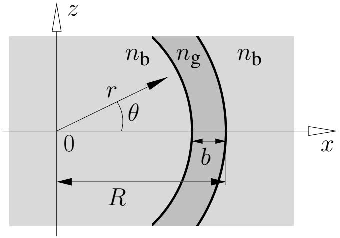
Modes of bent slab waveguides



$\sim \exp(i\omega t)$ (FD)

2-D TE / TM, cylind. coord. r, θ, y,
$\partial_y n = \partial_\theta n = 0$

Modes of bent slab waveguides



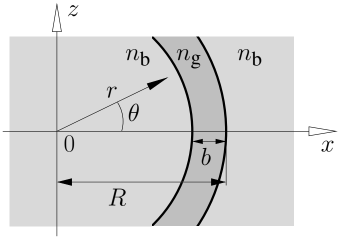
$\sim \exp(\mathrm{i}\omega t)$ (FD)

2-D TE / TM, cylind. coord. r, θ, y,
$\partial_y n = \partial_\theta n = 0$

$$\hookrightarrow \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} (r, \theta) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix} (r)\, \mathrm{e}^{-\mathrm{i}\gamma R \theta},$$

bent slab mode $\{\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}, \gamma = \beta - \mathrm{i}\alpha\}$.

Modes of bent slab waveguides



$\sim \exp(\mathrm{i}\omega t)$ (FD)

2-D TE / TM, cylind. coord. r, θ, y,
$\partial_y n = \partial_\theta n = 0$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}\gamma R\theta},$$

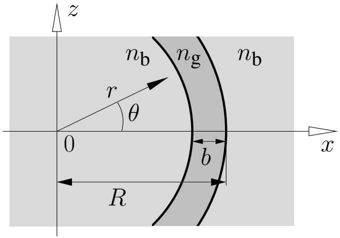
bent slab mode $\{\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}, \gamma = \beta - \mathrm{i}\alpha\}$.

- Piecewise constant $n(r)$, $\phi = \bar{E}_y$ (TE), $\phi = \bar{H}_y$ (TM)

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \left(k^2 n^2 - \frac{\gamma^2 R^2}{r^2}\right)\phi = 0\,,$$

(Bessel differential equation with (complex) order γR.)

Modes of bent slab waveguides



$\sim \exp(i\omega t)$ (FD)

2-D TE / TM, cylind. coord. r, θ, y,
$\partial_y n = \partial_\theta n = 0$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(r)\, e^{-i\gamma R\theta},$$

bent slab mode $\{\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}, \gamma = \beta - i\alpha\}$.

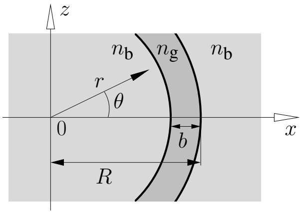
- Piecewise constant $n(r)$, $\phi = \bar{E}_y$ (TE), $\phi = \bar{H}_y$ (TM)

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \left(k^2n^2 - \frac{\gamma^2R^2}{r^2}\right)\phi = 0\,,$$

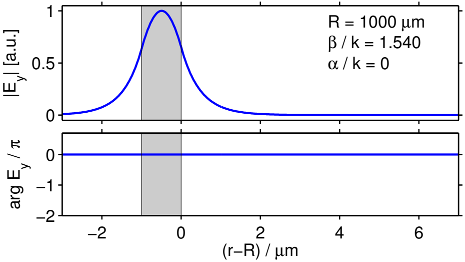
(Bessel differential equation with (complex) order γR.)

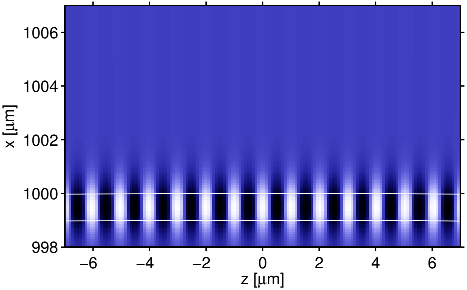
- Nonzero solutions,
- bounded at the origin, $\sim J_{\gamma R}(nkr)$ for $r < R - b$,
- outgoing exterior fields, $\sim H^{(2)}_{\gamma R}(nkr)$ for $r > R$, ($\sim \exp(i\omega t)$),
- continuity at interfaces : $\phi, \partial_r\phi$ (TE), $\phi, (\partial_r\phi)/n^2$ (TM).

Bend modes, 2-D examples

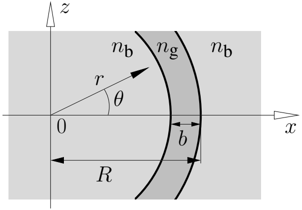


2-D, TE,
$n_b = 1.45$, $n_g = 1.60$, $b = 1.0\,\mu m$, $\lambda = 1.55\,\mu m$,
$R = 1000\,\mu m$.

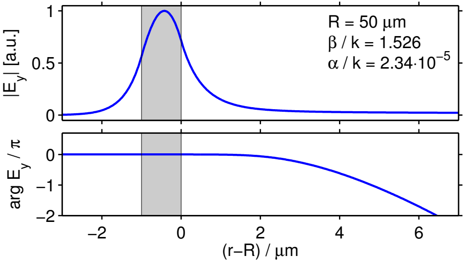


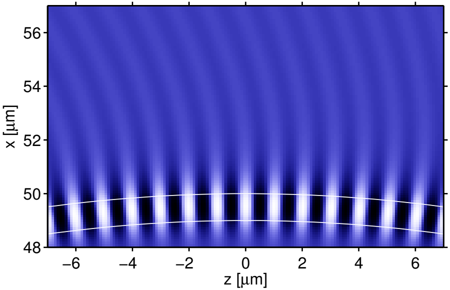


Bend modes, 2-D examples

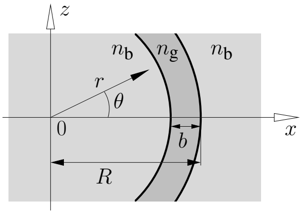


2-D, TE,
$n_b = 1.45$, $n_g = 1.60$, $b = 1.0\,\mu m$, $\lambda = 1.55\,\mu m$,
$R = 50\,\mu m$.

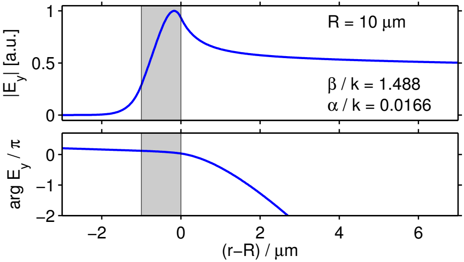


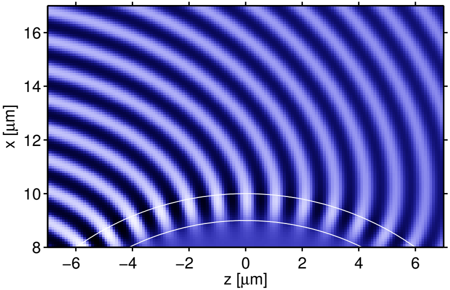


Bend modes, 2-D examples

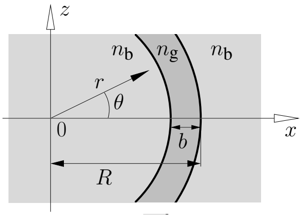


2-D, TE,
$n_b = 1.45$, $n_g = 1.60$, $b = 1.0\,\mu m$, $\lambda = 1.55\,\mu m$,
$R = 10\,\mu m$.

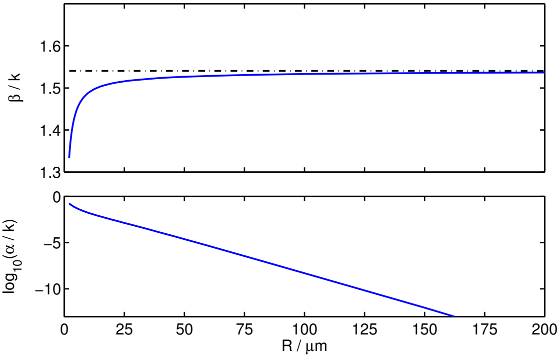




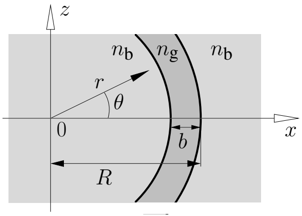
Propagation constant vs. bend radius



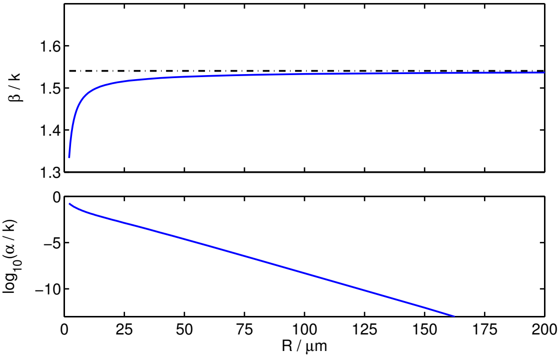
2-D, TE,
$n_b = 1.45$, $n_g = 1.60$, $b = 1.0\,\mu\text{m}$, $\lambda = 1.55\,\mu\text{m}$,
$R \in [2, 200]\,\mu\text{m}$.



Propagation constant vs. bend radius



2-D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = 1.0\,\mu\text{m},\ \lambda = 1.55\,\mu\text{m},$
$R \in [2, 200]\,\mu\text{m}.$

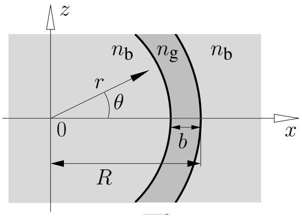


Alternative definition :
$R' = R - b/2.$

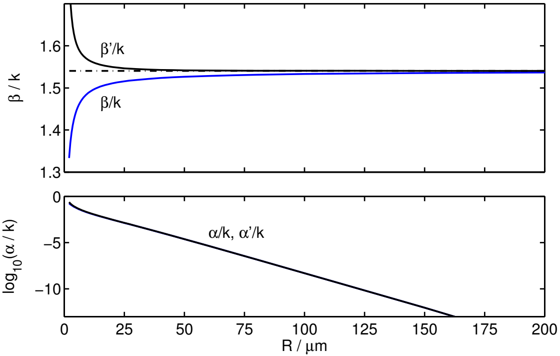
Identical physical fields

$$\gamma' R' = \gamma R,$$
$$\gamma' = \gamma\,\frac{R}{R - b/2}\,.$$

Propagation constant vs. bend radius



2-D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = 1.0\,\mu m,\ \lambda = 1.55\,\mu m,$
$R \in [2, 200]\,\mu m.$

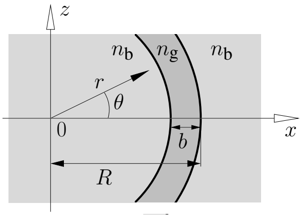


Alternative definition :
$R' = R - b/2.$

Identical physical fields

$$\gamma' R' = \gamma R,$$
$$\gamma' = \gamma \, \frac{R}{R - b/2} \, .$$

Power & orthogonality



2-D TE / TM bend modes:

- Power flow: $S_r \neq 0$, $S_r, S_\theta \sim \mathrm{e}^{-2\alpha R\theta}$, $S_\theta \sim |\phi|^2/r$

  ↪ $\int_0^\infty S_\theta(r)\,\mathrm{d}r < \infty$ ⟷ power normalization.

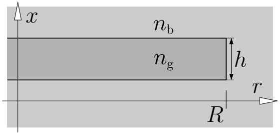
- Orthogonality of nondegenerate bend modes, product

$$[\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2] = \int_0^\infty (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \boldsymbol{e}_\theta \,\mathrm{d}r.$$

(Here [, ; ,] is complex valued.)

(Expressions $\sim \phi^2/r$ ⟷ convergence of the integrals.)

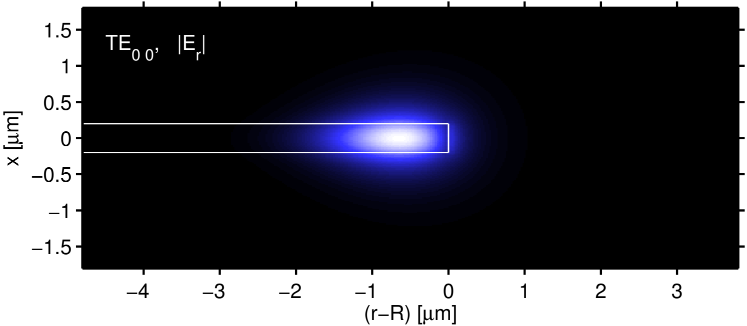
Bend modes supported by an angular disc segment



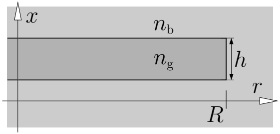
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

$TE_{0\,0}$
$\beta/k = 1.634$
$\alpha/k = 3.1 \cdot 10^{-8}$
[JCMwave].



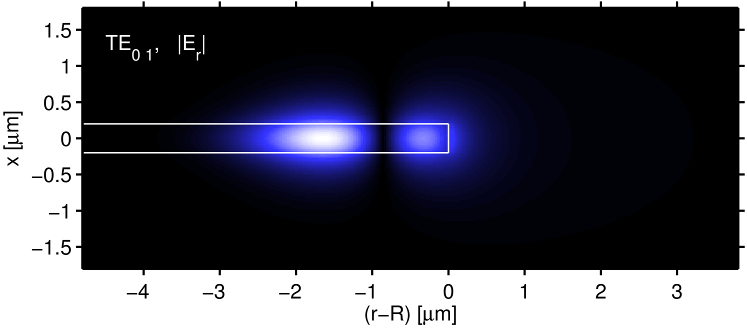
Bend modes supported by an angular disc segment



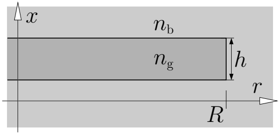
$\lambda = 1.55\ \mu m$,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4\ \mu m$,
$R = 20\ \mu m$;

$x \in [-3, 3]\ \mu m$,
$(r - R) \in [-8, 4]\ \mu m$;

$TE_{0\,1}$
$\beta/k = 1.548$
$\alpha/k = 1.5 \cdot 10^{-5}$
[JCMwave].



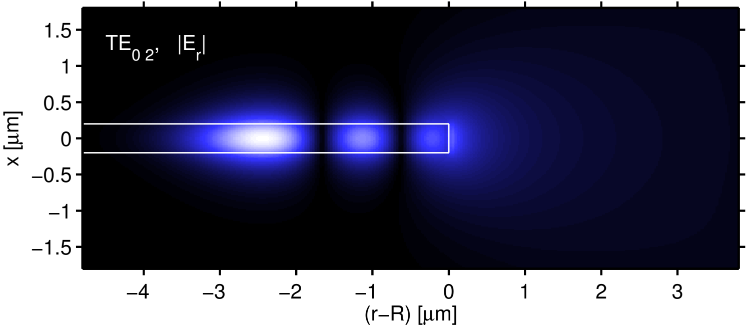
Bend modes supported by an angular disc segment



$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

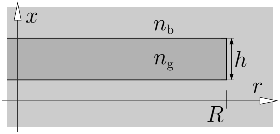
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$TE_{0\,2}$
$\beta/k = 1.480$
$\alpha/k = 4.0 \cdot 10^{-4}$
[JCMwave].



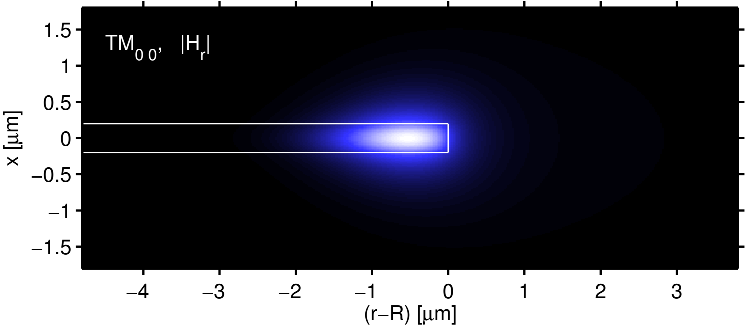
Bend modes supported by an angular disc segment

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

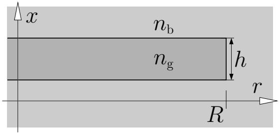


$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$TM_{0\,0}$
$\beta/k = 1.551$
$\alpha/k = 2.4 \cdot 10^{-5}$
[JCMwave].



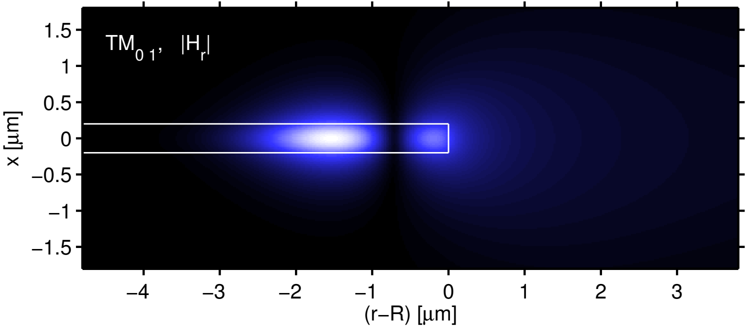
Bend modes supported by an angular disc segment



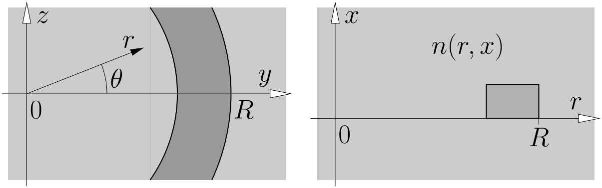
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

$TM_{0\,1}$
$\beta/k = 1.468$
$\alpha/k = 7.6 \cdot 10^{-4}$
[JCMwave].

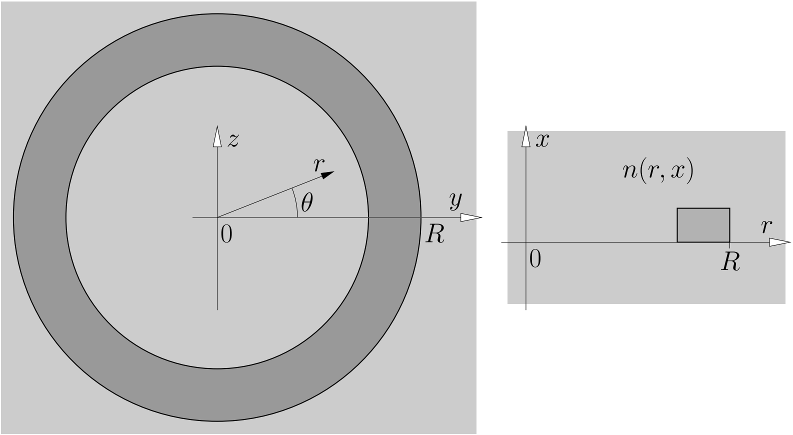


Circular microcavity



Bend modes

Circular microcavity

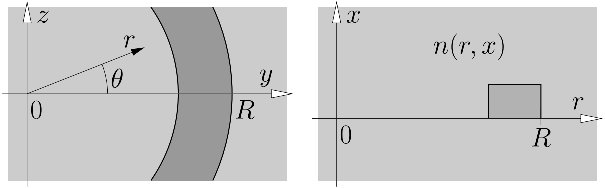


Bend modes ⇝ Whispering gallery resonances.

(Terms not always clearly distinguished.)

Whispering gallery resonances

~~(FD)~~



- Full cavity, $\theta \in [0, 2\pi]$:
  Look for resonances in the form of whispering gallery modes

$$\hookrightarrow \quad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} (r, \theta, x, t) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix} (r, x)\, \mathrm{e}^{\mathrm{i}\omega_{\mathrm{c}} t \,-\, \mathrm{i} m\theta},$$

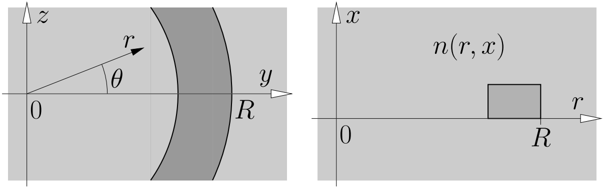
+c.c.

$\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}$: WGM profile, components $\tilde{E}_r, \tilde{E}_\theta, \tilde{E}_x, \tilde{H}_r, \tilde{H}_\theta, \tilde{H}_x$,
$m \in \mathbb{Z}$: angular order,
$\omega_{\mathrm{c}} = \omega'_{\mathrm{c}} + \mathrm{i}\omega''_{\mathrm{c}} \in \mathbb{C}$: eigenfrequency, $\omega'_{\mathrm{c}}, \omega''_{\mathrm{c}} \in \mathbb{R}$.

Q-factor $Q = \omega'_{\mathrm{c}}/(2\omega''_{\mathrm{c}})$, resonance wavelength $\lambda_{\mathrm{r}} = 2\pi \mathrm{c}/\omega'_{\mathrm{c}}$, outgoing radiation, FWHM: $2\delta\lambda = \lambda_{\mathrm{r}}/Q$.

Whispering gallery resonances

~~(FD)~~



- Piecewise constant $n(r,x)$, $\psi \in \{\tilde{E}_r, \tilde{E}_\theta, \tilde{E}_x, \tilde{H}_r, \tilde{H}_\theta, \tilde{H}_x\}$,

(Dispersion ?)

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \left(\frac{\omega_{\mathrm{c}}^2}{\mathrm{c}^2}n^2 - \frac{m^2}{r^2}\right)\psi = 0, \qquad \text{where } \partial n = 0,$$

& continuity conditions at interfaces (cylindrical coordinates),

& boundary conditions:
regularity at $r = 0$, outgoing waves at $x = \pm\infty$, $r = \infty$.

(or: normalizability versus x.)

Vectorial eigenproblem for whispering gallery resonances.

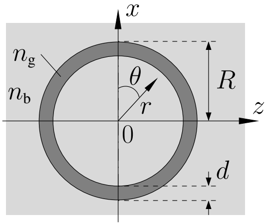
(Practical setting: computational domain $r_{\mathrm{i}} < r < r_{\mathrm{o}}$, $x_{\mathrm{b}} < x < x_{\mathrm{t}}$, PML boundary conditions / $\psi = 0$ at $r = r_{\mathrm{i}}$.)

2-D whispering gallery resonances

… as discussed for the 2-D TE / TM bend modes.

(WGMs: Bessel differential equation of integer order.)
(Notation: WGM(ρ, m) — mode of radial order ρ and angular order m.)

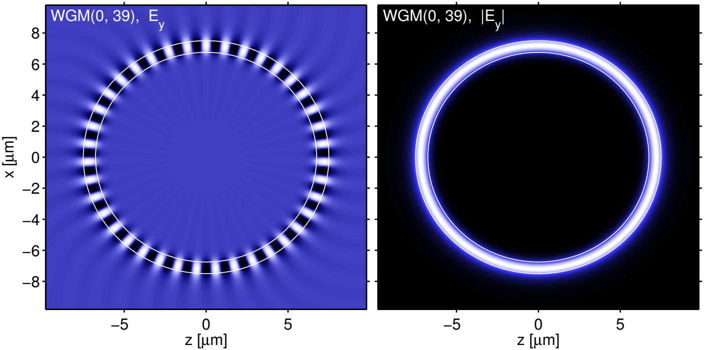
2-D whispering gallery resonances



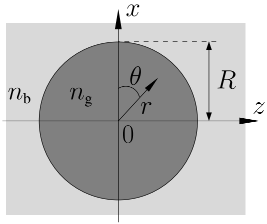
TE, $R = 7.5$ µm, $d = 0.75$ µm, $n_g = 1.5$, $n_b = 1.0$.

WGM(0, 39):

$\lambda_r = 1.5637$ µm, $Q = 1.1 \cdot 10^5$, $2\delta\lambda = 1.4 \cdot 10^{-5}$ µm.



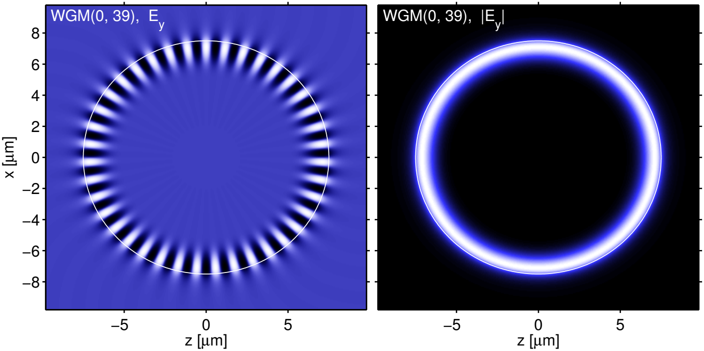
2-D whispering gallery resonances



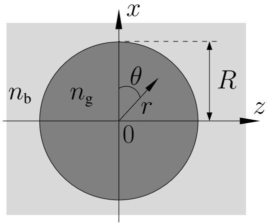
TE, $R = 7.5\,\mu\text{m}$, $n_g = 1.5$, $n_b = 1.0$.

WGM(0, 39):

$\lambda_r = 1.6025\,\mu\text{m}$, $Q = 5.7 \cdot 10^5$, $2\delta\lambda = 2.8 \cdot 10^{-6}\,\mu\text{m}$.



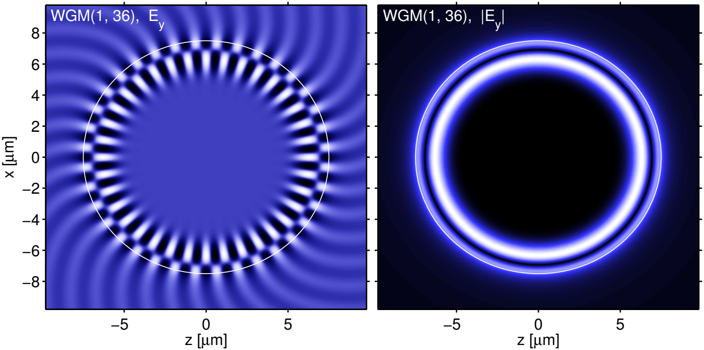
2-D whispering gallery resonances



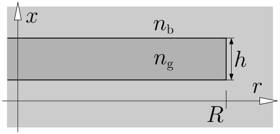
TE, $R = 7.5\,\mu\text{m}$, $n_g = 1.5$, $n_b = 1.0$.

WGM$(1, 36)$:

$\lambda_r = 1.5367\,\mu\text{m}$, $Q = 2.2 \cdot 10^3$, $2\delta\lambda = 7.0 \cdot 10^{-4}\,\mu\text{m}$.



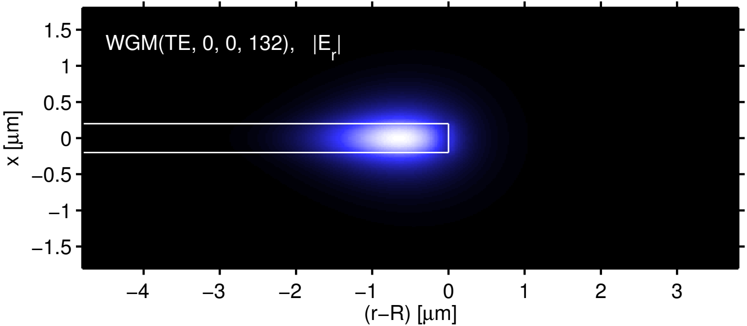
WGMs supported by a circular slab disc



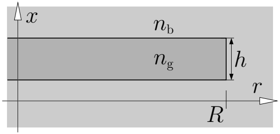
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

WGM(TE, 0, 0, 132)
$\lambda_r = 1.555$ µm
$Q = 6.9 \cdot 10^6$
[JCMwave].



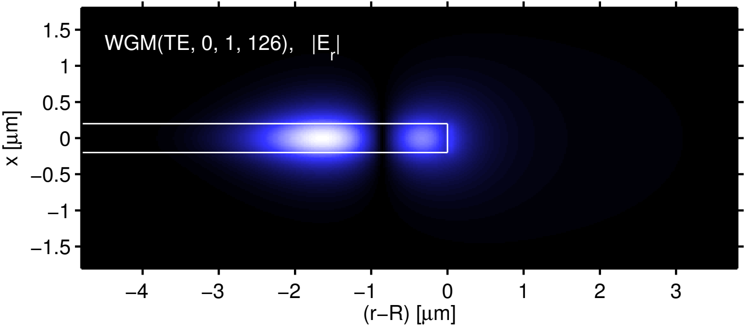
WGMs supported by a circular slab disc



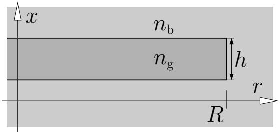
$\lambda = 1.55\ \mu m$,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4\ \mu m$,
$R = 20\ \mu m$;

$x \in [-3, 3]\ \mu m$,
$(r - R) \in [-8, 4]\ \mu m$;

WGM(TE, 0, 1, 126)
$\lambda_r = 1.545\ \mu m$
$Q = 1.7 \cdot 10^4$
[JCMwave].



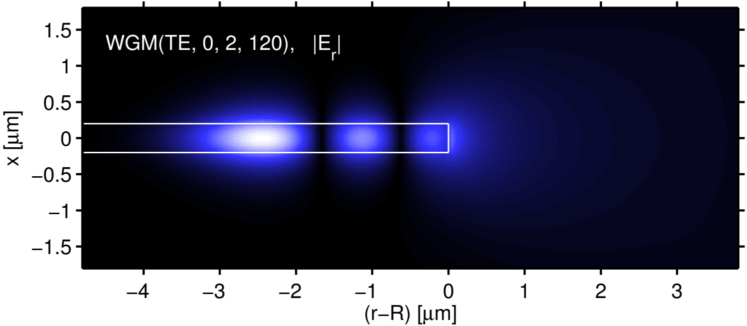
WGMs supported by a circular slab disc



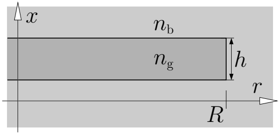
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

WGM(TE, 0, 2, 120)
$\lambda_r = 1.550$ µm
$Q = 5.7 \cdot 10^2$
[JCMwave].



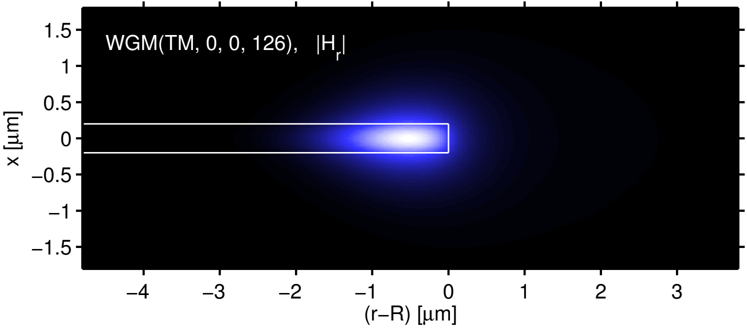
WGMs supported by a circular slab disc



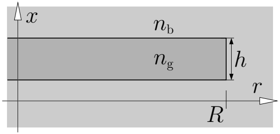
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

WGM(TM, 0, 0, 126)
$\lambda_r = 1.547$ µm
$Q = 1.0 \cdot 10^4$
[JCMwave].



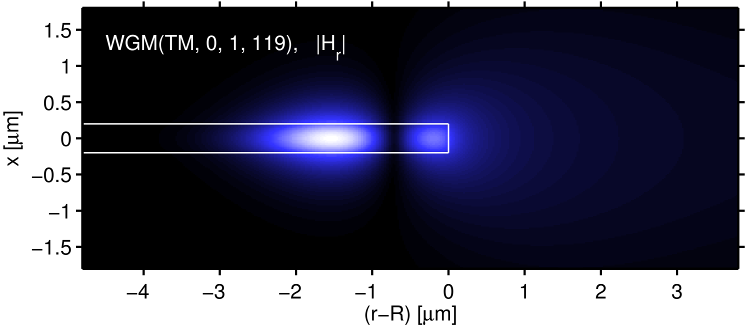
WGMs supported by a circular slab disc



$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

WGM(TM, 0, 1, 119)
$\lambda_r = 1.550$ µm
$Q = 3.0 \cdot 10^2$
[JCMwave].



Bend modes versus whispering gallery resonances

(Field supported by a full circular cavity.)
(Incompatible models, in principle.)

[BWG] $\omega \in \mathbb{R}$ given, $\gamma = \beta - \mathrm{i}\alpha \in \mathbb{C}$ eigenvalue,

$$\Phi(r, \theta, t) = \phi(r)\ \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta R\theta}\ \mathrm{e}^{-\alpha R\theta}.$$

[WGM] $\omega_{\mathrm{c}} = \omega_{\mathrm{c}} + \mathrm{i}\omega''_{\mathrm{c}} \in \mathbb{C}$ eigenvalue, $m \in \mathbb{Z}$ given,

$$\Psi(r, \theta, t) = \psi(r)\ \mathrm{e}^{\mathrm{i}\omega'_{\mathrm{c}} t - \mathrm{i}m\theta}\ \mathrm{e}^{-\omega''_{\mathrm{c}} t}.$$

Look at a resonant low-loss configuration:

- Translate $\omega \approx \omega'_{\mathrm{c}}$, $m \approx \beta R$.
- Equate the power loss during one time period $T = 2\pi/\omega \approx 2\pi/\omega'_{\mathrm{c}}$
  $\rightsquigarrow$ $\beta/\alpha \approx \omega'_{\mathrm{c}}/\omega''_{\mathrm{c}} = 2\,Q$.

Upcoming

Next lectures:

- Coupled mode theory, perturbation theory.
- Hybrid analytical / numerical coupled mode theory.
- A touch of photonic crystals; a touch of plasmonics.